

Esercizi svolti

Identità

Esercizio n.1

$$\frac{\text{sen}(x+y)}{\text{sen}x+\text{sen}y} = \frac{\text{sen}x-\text{sen}y}{\text{sen}(x-y)}$$

sapendo che $\text{sen}^2x - \text{sen}^2y = \text{sen}(x+y)\text{sen}(x-y)$
 si ha:

$$\begin{aligned} \frac{\text{sen}(x+y)}{\text{sen}x+\text{sen}y} \cdot \frac{\text{sen}x-\text{sen}y}{\text{sen}x-\text{sen}y} &= \frac{\text{sen}x-\text{sen}y}{\text{sen}(x-y)} \\ \frac{\text{sen}(x+y)(\text{sen}x-\text{sen}y)}{\text{sen}^2x-\text{sen}^2y} &= \frac{\text{sen}x-\text{sen}y}{\text{sen}(x-y)} \\ \frac{\text{sen}(x+y)(\text{sen}x-\text{sen}y)}{\text{sen}(x+y)\text{sen}(x-y)} &= \frac{\text{sen}x-\text{sen}y}{\text{sen}(x-y)} \\ \frac{(\text{sen}x-\text{sen}y)}{\text{sen}(x-y)} &= \frac{\text{sen}x-\text{sen}y}{\text{sen}(x-y)} \end{aligned}$$

infatti:

$$\begin{aligned} (\text{sen}x\text{cos}y + \text{sen}y\text{cos}x)(\text{sen}x\text{cos}y - \text{sen}y\text{cos}x) &= \\ = \text{sen}^2x\text{cos}^2y - \text{sen}^2y\text{cos}^2x &= \\ = \text{sen}^2x(1 - \text{sen}^2y) - \text{sen}^2y(1 - \text{sen}^2x) &= \\ = \text{sen}^2x - \text{sen}^2x\text{sen}^2y - \text{sen}^2y + \text{sen}^2y\text{sen}^2x &= \\ = \text{sen}^2x - \text{sen}^2y \text{ c.v.d.} \end{aligned}$$

Oppure, metodo più lungo, moltiplichiamo 1° e 2° membro per $\frac{\text{sen}(x+y)}{\text{sen}x+\text{sen}y}$ e

Esercizio n.2

$$\text{cosec}(\alpha - \beta) = \frac{\text{sen}\alpha\text{cos}\beta + \text{sen}\beta\text{cos}\alpha}{\text{sen}^2\alpha - \text{sen}^2\beta}$$

$$\begin{aligned} \frac{1}{\text{sen}(\alpha - \beta)} &= \frac{\text{sen}\alpha\text{cos}\beta + \text{sen}\beta\text{cos}\alpha}{\text{sen}^2\alpha - \text{sen}^2\beta} \\ \frac{1}{\text{sen}(\alpha - \beta)} &= \frac{\text{sen}(\alpha + \beta)}{\text{sen}(\alpha + \beta)\text{sen}(\alpha - \beta)} \\ \frac{1}{\text{sen}(\alpha - \beta)} &= \frac{1}{\text{sen}(\alpha - \beta)} \end{aligned}$$

Oppure, metodo più lungo, moltiplichiamo 1° e 2° membro per $\text{sen}(\alpha - \beta)$ e poi ...